

Freeze-out conditions for production of resonances, hadronic molecules, and light nuclei

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We investigate the freeze-out conditions of a particle in an expanding system of interacting particles in order to understand the productions of resonances, hadronic molecules and light nuclei in heavy ion collisions. Applying the kinetic freeze-out condition with explicit hydrodynamic calculations for the expanding hadronic phase to the daughter particles of K^* mesons, we find that the larger suppression of the yield ratio of K^*/K at LHC than at RHIC compared to the expectations from the statistical hadronization model based on chemical freeze-out parameters reflects the lower kinetic freeze-out temperature at LHC than at RHIC. Furthermore, we point out that for the light nuclei or hadronic molecules that are bound, the yields are affected by the freeze-out condition of the respective particle in the hadronic matter, which leads to the observation that the deuteron production yields are independent of the size of deuteron, and depend only on the number of ground state constituents.

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Relativistic heavy ion collision experiments have enriched our understanding on the properties of quantum chromodynamic (QCD) matter at high temperature and densities [1–3]. One of the important experimental objectives in relativistic heavy ion collision experiments is confirming the occurrence of the phase transition between a system composed of free quarks and gluons, the so-called quark-gluon plasma (QGP), and the hadronic matter that was predicted by Lattice QCD calculations [4]. In that respect, the large numbers of various kinds of hadronic particles produced at the phase boundary have been useful in elucidating the entire process of heavy ion collisions, since production yields of hadron are affected by conditions at each step of the system evolution.

The systematic studies on the hadron production yields using the statistical hadronization model have led to the identification of conditions at the phase boundary; the phase transition temperature and the baryon chemical potential [5–8]. At the same time among many measurements of the hadron yields from heavy ion collision, those of the light nuclei are remarkable. This is so because light nuclei are composed of nucleons produced at chemical freeze-out and hence expected to be produced some time after the chemical freeze-out. On the other hand, the yields of light nuclei are explainable in the statistical hadronization model with the same conditions at which normal hadrons are produced. Another puzzle is the production yield ratio of K^*/K , which is overestimated by the statistical model [8]. Moreover, the ratio is smaller at Large Hadron Collider (LHC) than at Relativistic Heavy Ion Collider (RHIC) [9, 10].

In general, production yields of hadronic molecules, light nuclei or resonances not only depend on the conditions at the chemical freeze-out but also on their inter-

actions with other hadrons during the hadronic stage. Specifically, for the light nuclei or hadronic molecules that are bound, the yields are affected by the freeze-out condition of the respective particle in the hadronic matter. For resonances that decay into daughter particles, the freeze-out point of the daughter particles will be important as the resonances are reconstructed from the observed daughter particles. In other words, investigation on the yields of hadronic molecules or resonances may result in understanding of how long the hadronic stage lasts in heavy ion collisions, or of when the kinetic freeze-out of various particles takes place. In this letter, we focus on the freeze-out conditions and discuss natural consequences that follow from applying these conditions to understand the production yields of resonances and hadronic molecules.

Kinetic freeze-out of a particle species i from the matter occurs when its scattering rate τ_{scatt}^i becomes larger than the expansion rate of the system τ_{exp} [11]. Hence, in an expanding system of interacting particles, freeze-out takes place when

$$\tau_{exp} = \frac{1}{\partial \cdot u} = \tau_{scatt}^i = \frac{1}{\sum_j \langle \sigma_{ij} v_{ij} \rangle n_j}, \quad (1)$$

with $\langle \sigma_{ij} v_{ij} \rangle$ being the thermally averaged cross section times the relative velocity between particle species i and j , n_j the density of particle j , and u the expansion velocity of the system.

The expansion time τ_{exp} can be approximated to the ratio of the fireball volume V to the change of that in time, $V/(dV/dt)$, which can be further reduced to $R/(3dR/dt)$ for the spherically expanding fireball with its radius R . Let us for simplicity further assume that the system is composed of one species only and that the

cross section is velocity independent. The freeze-out condition in Eq. (1) then becomes

$$\frac{R}{3dR/dt} = \frac{1}{n\sigma\langle v \rangle}. \quad (2)$$

In general the relation between dR/dt and $\langle v \rangle$ is not universal; particularly so when there is a flow. However, assuming that the rate of change in the radius is close to the average velocity of the particles in the system, the condition for the kinetic freeze-out becomes

$$\frac{N}{R_{fo}^2} = \frac{4\pi}{\sigma_{fo}}, \quad (3)$$

where a subscript $_{fo}$ stands for physical quantities at kinetic freeze-out and N is the total number of particles. We see that the two dimensional density determines the condition for freeze-out, because the transverse total cross section determines whether the particle interacts with the medium when it escapes from the medium.

On the other hand, the three dimensional density at freeze-out then goes as

$$\frac{N}{R_{fo}^3} = \left(\frac{4\pi}{\sigma_{fo}} \right)^{3/2} \frac{1}{N^{1/2}}. \quad (4)$$

This suggests that for higher collision energies and/or when the initial temperature and/or the number of particles increases, the three dimensional density at which freeze-out takes places becomes smaller. Let us discuss the relevance of this result in understanding the K^* and/or resonance production in heavy ion collisions.

It has been known that the experimentally measured yield of K^* mesons does not agree with the statistical hadronization model prediction [8–10]. Due to the short lifetime of the K^* meson compared to the lifespan of the hadronic stage in heavy ion collisions, K^* mesons not only participate in the hadronic interactions with light mesons but also decay to and reform from kaons and pions during the hadronic stage. As the daughter particles of K^* mesons are subject to re-scatter as well in the hadronic medium, the measured K^* meson reconstructed from an invariant mass analysis of the daughter particles will reflect its number when the daughter particles freeze out from the medium.

In order to investigate the hadronic effects on the K^* meson abundance in the hadronic medium in general, we consider simplified rate equations for the abundances of both K^* and K mesons during the hadronic stage,

$$\begin{aligned} \frac{dN_{K^*}(\tau)}{d\tau} &= \frac{1}{\tau_{scatt}^K} N_K(\tau) - \frac{1}{\tau_{scatt}^{K^*}} N_{K^*}(\tau), \\ \frac{dN_K(\tau)}{d\tau} &= \frac{1}{\tau_{scatt}^{K^*}} N_{K^*}(\tau) - \frac{1}{\tau_{scatt}^K} N_K(\tau), \end{aligned} \quad (5)$$

with $1/\tau_{scatt}^{K^*} = \sum_i \langle \sigma_{K^*i} v_{K^*i} \rangle n_i$, and $1/\tau_{scatt}^K = \sum_j \langle \sigma_{Kj} v_{Kj} \rangle n_j$, Eq. (1). Here i, j stands for mostly

the light mesons such as pions and ρ mesons, i.e., $1/\tau_{scatt}^{K^*} = \langle \sigma_{K^*\rho \rightarrow K\pi} v_{K^*\rho} \rangle n_\rho + \langle \sigma_{K^*\pi \rightarrow K\rho} v_{K^*\pi} \rangle n_\pi + \langle \Gamma_{K^*} \rangle$, $1/\tau_{scatt}^K = \langle \sigma_{K\pi \rightarrow K^*\rho} v_{K\pi} \rangle n_\pi + \langle \sigma_{K\rho \rightarrow K^*\pi} v_{K\rho} \rangle n_\rho + \langle \sigma_{K\pi \rightarrow K^*} v_{K\pi} \rangle n_\pi$ with $\langle \Gamma_{K^*} \rangle$ being the thermally averaged decay width of the K^* meson [12]. Here we do not consider non-linear terms originated from the interaction between K^* mesons or kaons, like $K\bar{K} \rightarrow \rho\pi$.

While the exact solution of Eq. (5) including the non-linear terms was obtained in Ref. [12], the essential physics can be understood in the following simple limit. When the thermal cross sections and densities of light mesons are independent of time, the following analytic solution for the yield ratio between K^* mesons and kaons, $R = N_{K^*}/(N_K + N_{K^*})$ can be obtained from the above coupled differential equations,

$$R(\tau) = R_0 + \left(\frac{N_{K^*}^0}{N^0} - \frac{\tau_{scatt}}{\tau_{scatt}^K} \right) e^{-\frac{\tau - \tau_h}{\tau_{scatt}}}. \quad (6)$$

with $R_0 = \tau_{scatt}/\tau_{scatt}^K$ and $\tau_{scatt} = \tau_{scatt}^{K^*}\tau_{scatt}^K/(\tau_{scatt}^{K^*} + \tau_{scatt}^K)$. The initial yields for both hadrons are assumed to be N_K^0 and $N_{K^*}^0$ and $N^0 = N_K^0 + N_{K^*}^0$. In Eq. (6) the first term R_0 represents the abundance ratio of K^* mesons to kaon at the equilibrium temperature, while the second term through τ_{scatt} determines the rate at which the new equilibrium number is reached. If the τ_{scatt} is small, the abundance reaches the equilibrium ratio instantly.

It has been found that the actual variation of the abundance ratio of K^* mesons to kaons during the hadronic stage shows similar features as the solution, Eq. (6) but with a decreasing equilibrium ratio R_0 due to the decreasing background temperature [12]. Since the transient term in the yield ratio $R(\tau)$ plays a negligible role at a later time during the hadronic interaction stage, the equilibrium ratio, $R_0 = \tau_{scatt}/\tau_{scatt}^K$ that reflects the thermal K^*/K ratio of the hadron gas at the background temperature, mainly determines the ratio of K^* mesons to kaons towards the end of the hadronic stage.

The $R_0 = \tau_{scatt}/\tau_{scatt}^K$ decreases as the system expands and the temperature of the hadron gas decreases. Therefore, the final ratio between K^* and K mesons may reflect the condition at the last stage of the hadronic effects on K^* and K mesons, the kinetic freeze-out temperature. The K^* meson is measured in experiment by reconstructing the invariant mass of K and π mesons. If the freeze-out times for K^* , K and π mesons are same, then the measured K^*/K yield will reflect the temperature at the single freeze-out temperature. However, if the freeze-out time of π mesons is later than those of K^* and K mesons, due to the larger cross section with the medium, the part of K^* mesons will be lost in the reconstruction, because the additional scattering of pions (after the freeze-out of K^* mesons) will wash out the origin of pions. Therefore, the experimentally measured K^*/K ratio will reflect the freeze-out temperature of π mesons.

The reduction of the K^* meson yield from the statistical hadronization model expectation is found to be more

significant at LHC than at RHIC. As discussed before, the ratio of K^* to K mesons reflects the condition at the last stage of the K^* meson interaction in the hadronic medium so that the reduction of the ratio between K^* mesons and kaons at LHC originates from the lower kinetic freeze-out temperature of the system at LHC compared to that at RHIC.

The idea of the lower kinetic freeze-out temperature at LHC compared to that at RHIC becomes evident if we recall the density of particles at the kinetic freeze-out in heavy ion collisions, Eq. (4). That is, the density at freeze-out becomes smaller when the number of initially produced particles increases. In the following, we will demonstrate that the freeze-out temperature for pions are lower at LHC than at RHIC by explicitly applying the condition in Eq. (1) to both cases.

First, we evaluate the expansion time of the system $\tau_{exp} = V/(dV/dt)$ for both at RHIC and at LHC using hydrodynamics simulations. Hydrodynamic equations are expressed as $\partial_\mu T^{\mu\nu} = 0$, where the energy-momentum tensor $T^{\mu\nu} = (e + p)u^\mu u^\nu - pg^{\mu\nu}$ with e , p and u^μ being, respectively, energy density, pressure, and 4-velocity of flow. Using $u_\nu \partial_\mu T^{\mu\nu} = 0$ and the first law of thermodynamics for infinitely small volume of cell [13],

$$\partial_\mu(eu^\mu) = T\partial_\mu(su^\mu) - p(\partial_\mu u^\mu), \quad (7)$$

with T and s being temperature and entropy density, we can derive the entropy conservation, $\partial_\mu(su^\mu) = 0$. For simplicity, we assume boost-invariance and consider central collisions, that is, symmetric expansion in transverse plane. Then there are only two independent hydrodynamic equations [14]:

$$\frac{1}{\tau}\partial_\tau(\tau T^{\tau\tau}) + \frac{1}{r}\partial_r(r T^{\tau r}) = \frac{p}{\tau}, \quad (8)$$

$$\frac{1}{\tau}\partial_\tau(\tau s u^\tau) + \frac{1}{r}\partial_r(r s u^r) = 0, \quad (9)$$

where $\tau = \sqrt{t^2 - z^2}$ and $r = \sqrt{x^2 + y^2}$. Integrating the above equations over transverse plane, we have

$$\frac{1}{\tau}\partial_\tau(\tau \int dA T^{\tau\tau}) = \frac{1}{\tau} \int dA p, \quad (10)$$

$$\frac{1}{\tau}\partial_\tau(\tau \int dA s u^\tau) = 0, \quad (11)$$

where $dA = 2\pi r dr$. We note that the second terms in the left-hand side of Eq. (8) and (9) disappear due to boundary condition. Here we make the assumption that nuclear matter has definite boundary and e , s , and p are uniform inside. Then we have the following simple equations:

$$\frac{1}{\tau}\partial_\tau(\tau A \langle T^{\tau\tau} \rangle) = \frac{1}{\tau} p A, \quad \frac{1}{\tau}\partial_\tau(\tau A s \langle u^\tau \rangle) = 0, \quad (12)$$

where $A = \pi R^2$ with R being the radius of nuclear matter and $\langle T^{\tau\tau} \rangle = \int dA T^{\tau\tau}/A = (e + p)\langle \gamma_r^2 \rangle - p$, $\langle u^\tau \rangle = \langle \gamma_r \rangle$,

with $\gamma_r = 1/\sqrt{1 - v_r^2}$ and v_r being radial velocity. Assuming that the radial flow velocity is a linear function of the radial distance from the center, i.e., $\gamma_r v_r = \gamma_R \dot{R}(r/R)$, where $\dot{R} = \partial R/\partial \tau$ and $\gamma_R = 1/\sqrt{1 - \dot{R}^2}$,

$$\langle \gamma_r^2 \rangle = 1 + \frac{\gamma_R^2 \dot{R}^2}{2}, \quad \langle \gamma_r \rangle = \frac{2}{3\gamma_R^2 \dot{R}^2} (\gamma_R^3 - 1). \quad (13)$$

We then numerically solve Eq. (12) by using the lattice equation of state [15, 16]. The initial thermalization time for hydrodynamic simulations is assumed 0.5 fm/c, and the initial radius is given by the transverse area where local temperature is above 150 MeV. Though the hydrodynamic approach is marginal in hadron gas phase, it has successfully reproduced abundant experimental data from relativistic heavy-ion collisions [17, 18]. We show the results for LHC and RHIC as a function of both the proper time and the temperature in Fig. 1.

Second, we consider $1/\tau_{scatt} \approx \sigma_\pi \langle v \rangle n_\pi$. Taking the total cross section of the pion in the hadronic medium to be 40 mb, $\langle v \rangle = 0.7c$, and assuming that pions are in thermal equilibrium during the hadronic stage, $n_\pi(T) \approx \frac{g_\pi}{2\pi^2} m_\pi^2 T K_2(m_\pi/T)$, with g_π and m_π being the degeneracy and mass of pions, respectively, and K_2 the modified Bessel function of the second kind, we obtain $\tau_{scatt}^{\pi,a}$ shown in Fig. 1. In addition, we consider a numerically evaluated pion scattering time [19] parameterized as $1/\tau_{scatt}^\pi = (59.5 \text{ fm}^{-1})(T/1 \text{ GeV})^{3.45}$ [20]. In Fig. 1 we show the above parameterized scattering time multiplied by the phenomenological parameter $\xi = 0.295$ [20], denoted by $\tau_{scatt}^{\pi,b}$. When evaluating pion scattering times, we have used the temperature as a function of proper time, $T(\tau)$ obtained from hydrodynamics simulations.

As shown in Fig. 1 (a), both pion scattering times for RHIC, $\tau_{scatt}^{\pi,aR}$ and $\tau_{scatt}^{\pi,bR}$ cross the expansion time of the system earlier than that for LHC, $\tau_{scatt}^{\pi,aL}$ and $\tau_{scatt}^{\pi,bL}$, implying that the kinetic freeze-out temperature at RHIC is higher than that at LHC; Fig. 1 (b) clearly demonstrates this. The line of the pion scattering time crosses the expansion velocity of the system at RHIC before that at LHC. We read in the inset of Fig. 1 (b) that the kinetic freeze-out occurs at about 130 MeV at RHIC, and at about 120 MeV at LHC from $\tau_{scatt}^{\pi,a}$, or at about 135 MeV at RHIC, and at about 125 MeV at LHC from $\tau_{scatt}^{\pi,b}$. These values are consistent with the temperatures needed to explain the experimentally measured K^*/K ratio at RHIC and LHC with the solutions of Eq. (5) [12].

The above argument applies to any resonances or hadronic molecules that decay to certain daughter particles. It should be noted also that the yields of the resonances or hadronic molecules will decrease when the cross section of the daughter particles with the medium increases. At the same time, for strongly bound states such as the light nuclei, the freeze-out temperature will be determined by the cross section of the bound state itself with the medium.

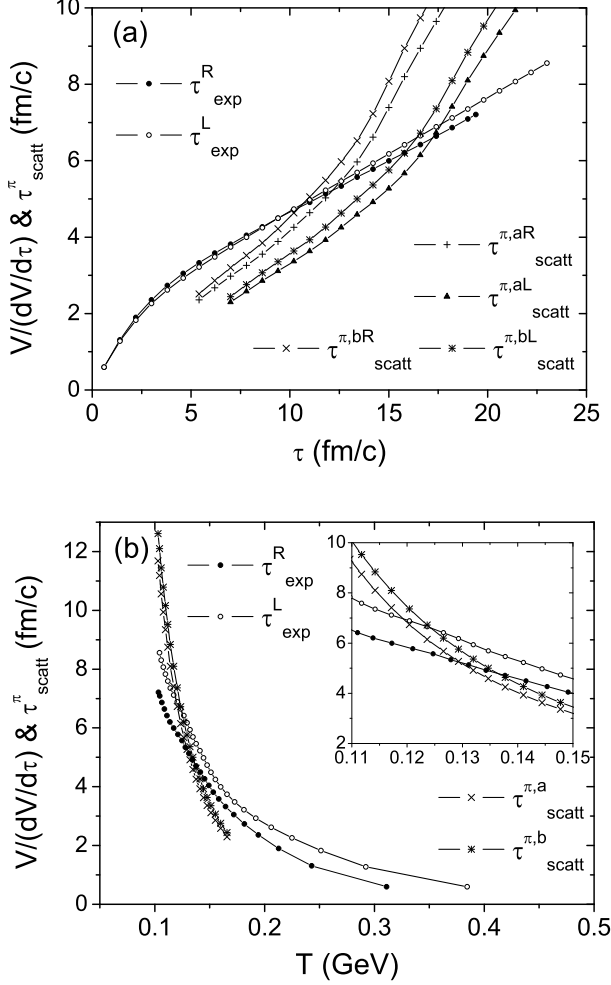


FIG. 1: Variations of the τ^{π}_{scatt} and the expansion time of the system, $\tau_{exp} = V/(dV/dt)$ at RHIC and LHC both in time (a) and temperature (b) during the hadronic stage.

Let us consider the coalescence production of the deuteron during the hadronic stage in heavy ion collisions. In the same model used in the evaluation of hadronic molecule states [21], the yield of deuteron is given by,

$$N_D = \frac{g_D}{g_N^2} \frac{N_N^2}{V_{fo}} \frac{(4\pi a_D^2)^{3/2}}{1 + 2m_N T_{fo} a_D^2}, \quad (14)$$

where g_D and g_N are the degeneracy of the deuteron and nucleon, respectively, N_N the number of nucleons, and m_N the mass of the nucleon. We represent the freeze-out volume at hadronization as V_{fo} and the hadronization temperature T_{fo} . a_D is the radius of the deuteron. Assuming that the size of the final bound state is related to the cross section of the state with the medium, the above

can be written again as,

$$N_D = \frac{g_D}{g_N^2} N_N^2 \frac{3}{4\pi} \left(4 \frac{\sigma_D}{R_{fo}^2} \right)^{3/2} \frac{1}{1 + 2m_N T_{fo} a_D^2} \\ = \frac{g_D}{g_N^2} N_N^2 \frac{48\pi^{1/2}}{N_\pi^{3/2}} \frac{1}{1 + 2m_N T_{fo} a_D^2}, \quad (15)$$

where we have assumed that $V_{fo} = 4\pi/3 R_{fo}^3$ and $\sigma_{fo} = \pi a_D^2 = \sigma_D$, the cross section of the deuteron, and have used the condition at the freeze-out, the relation between the cross section of the deuteron σ_{fo} and the freeze-out radius R_{fo} , Eq. (3) with pions as the dominant constituents of the medium. As we see, the coalescence production of the deuteron is almost independent of the size of the deuteron and only depends on the number of constituents of the medium. When the cross section is large, it freezes out later. Assuming that the number of ground state constituents do not change much during the hadronic phase, one could explain why the observed production yield of the deuteron is well understood in the statistical hadronization model.

We have shown that the yield ratio of K^*/K from heavy ion collisions at RHIC and at LHC can be understood by applying the kinetic freeze-out condition to the daughter particles. Since the ratio is mainly determined from the relative interaction ratio, $\tau_{scatt}/\tau_{scatt}^K$, which reflects temperature of the system, it is possible to infer the condition at the kinetic freeze-out. The larger suppression of the ratio at LHC than at RHIC compared to the expectations from statistical model based on chemical freeze-out parameters reflects the lower temperature at LHC as demonstrated explicitly by applying the freeze-out condition to hydrodynamic calculations.

Furthermore, we have pointed out that for the light nuclei or hadronic molecules that are bound, the yields are affected by the freeze-out condition of the respective particle with the hadronic matter. This led to the observation that the deuteron production yields are independent of the size of deuteron, and depend only on the number of ground state constituents. We therefore suggest that studying production of various hadrons such as hadronic molecules, resonances and light hadrons not only provides us chances to understand hadron formation in heavy ion collisions but also gives us valuable information on the evolution of the entire system itself.

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